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A perturbation solution for heating a rectangular sensible heat storage packed bed with a constant temperature at the walls

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Abstract—This paper presents an analytical solution for heating a rectangular sensible heat storage packed bed with a constant temperature at the walls by a non-thermal equilibrium flow of incompressible fluid. A two energy equation model is employed to simulate the temperature difference between the fluid and solid phases. Using the perturbation technique, an analytical solution for the problem is obtained. It is shown that the temperature difference between the fluid and solid phases consists of the steady and transient components. The steady component is localized near the inlet boundary. The transient component describes a wave propagating from the fluid inlet boundary. The amplitude of the wave quickly decreases while the wave propagates downstream. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

Packed beds are often used for the storage of heat energy and in the chemical industry. These important applications explain the permanent interest in the transport phenomena in packed beds for analytical and numerical investigations. Most of the analytical studies of the phenomena which utilize the non-thermal equilibrium approach were concentrated on the Schumann model of a packed bed suggested in ref. [1]. This model ignores the thermal conduction terms in both the fluid and solid phase energy equations. Analytical solutions for this model for various boundary conditions are obtained in refs. [2–7].

Numerical solutions for the non-thermal equilibrium model, which utilize the full energy equations for the fluid and solid phases, have been recently presented in refs. [8–11]. In these references a non-thermal equilibrium, forced fluid flow through a rectangular packed bed is considered. Two types of boundary conditions at the packed bed walls are considered : (a) a constant temperature boundary condition and (b) an insulated boundary condition. Proceeding from the results of comprehensive numerical calculations, ref. [10] shows that the 1-D formulation is sufficient in the case of an insulated boundary condition, while the 2-D formulation is necessary for the case of a constant wall temperature boundary condition.

In the present paper we present an analytical solution for the problem with the constant wall temperature boundary condition. In obtaining this solution the perturbation analysis suggested in ref. [12] is utilized. The price for obtaining the analytical solution is assuming, in addition to the assumptions made in refs. [8–11], that the fluid is incompressible, the flow rate at every cross section of the packed bed is constant and uniform, and properties of the fluid and solid phases are constant.

PHYSICAL MODEL AND GOVERNING EQUATIONS

Figure 1 depicts the schematic diagram of the problem under consideration. A 2-D porous packed bed is filled with the incompressible fluid and is initially at a uniform temperature. At the instant $t = 0$, fluid at a higher temperature is suddenly allowed to flow through the packed bed. The walls of the packed bed are kept at a given constant temperature during the process. In establishing a model for analyzing this problem, the following assumptions and simplications are employed :

- the fluid phase is incompressible, flow is uniform and the mass flow rate at every cross section of the packed bed is constant ;
- thermal, physical, and transport properties are constant ;
- heat transfer is 2-D and fluid flow is 1-D.

In ref. [12] it is shown that for many applications the product $h_{sf}a_{sf}$ is large and the dimensionless solid phase temperature differs from the fluid phase temperature by a small perturbation :

$$\Theta_s = \Theta_f + \delta\Delta\Theta \qquad (1)$$

where δ is a dimensionless small parameter. Following the perturbation analysis presented in ref. [12] it is easy to show that under these assumptions the dimen-

NOMENCLATURE

a_{sf}	specific surface area common to solid and fluid phases [m^2/m^3]
c_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
h_{sf}	fluid–particle heat transfer coefficient between solid and fluid phases [W m^{-2} K^{-1}]
L_i	length of the packed bed in x-direction (for $i = 1$) and in y-direction (for $i = 2$) [m]
R_i	dimensionless length of the packed bed (in ξ_i-direction)
t	time [s]
T	temperature [K]
v	velocity of the fluid phase [m s^{-1}]
x, y	Cartesian coordinates [m].

Greek letters

δ	dimensionless small parameter
ε	porosity
λ	thermal conductivity [W m^{-1} K^{-1}]
Θ	dimensionless temperature
$\Delta\Theta$	dimensionless difference between temperatures of the fluid and solid phases
ρ	density [kg m^{-3}]
τ	dimensionless time
ξ_i	dimensionless coordinates.

Subscripts

eff	effective property
in	inlet
f	fluid
0	initial
s	solid
w	wall.

sionless fluid phase temperature is governed by the following equation

$$\frac{\partial \Theta_f}{\partial \tau} + \frac{\partial \Theta_f}{\partial \xi_1} = \frac{\partial^2 \Theta_f}{\partial \xi_1^2} + \frac{\partial^2 \Theta_f}{\partial \xi_2^2} \tag{2}$$

and the dimensionless temperature difference between the solid and fluid phases is governed by the following equation

$$\Delta\Theta = (1-\Lambda_2)\frac{\partial \Theta_f}{\partial \tau} + (\Lambda_1 - \Lambda_2)\frac{\partial \Theta_f}{\partial \xi_1} \tag{3}$$

In equations (1)–(3) the following dimensionless variables are utilized:

temperature

$$\Theta = \frac{T - T_w}{T_{in} - T_w}$$

where T_{in} is the inlet temperature of the fluid phase and T_w is the temperature at the walls of the packed bed,

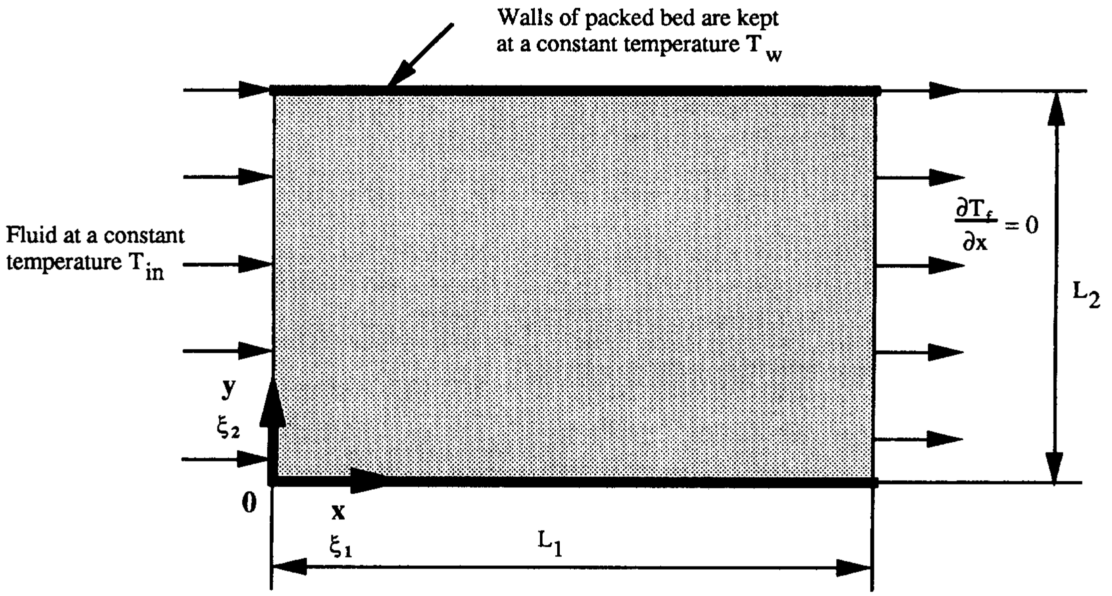

Fig. 1. Schematic diagram of the porous packed bed.

coordinates

$$\xi_1 = \frac{\rho_f(c_p)_f v_f}{\lambda_{feff}+\lambda_{seff}} x \quad \text{and} \quad \xi_2 = \frac{\rho_f(c_p)_f v_f}{\lambda_{feff}+\lambda_{seff}} y$$

time

$$\tau = \frac{[\rho_f(c_p)_f v_f]^2}{[\varepsilon\rho_f(c_p)_f+(1-\varepsilon)\rho_s(c_p)_s](\lambda_{feff}+\lambda_{seff})} t$$

and the dimensionless parameters

$$\Lambda_1 = \frac{\varepsilon\rho_f(c_p)_f+(1-\varepsilon)\rho_s(c_p)_s}{\varepsilon\rho_f(c_p)_f}$$

$$\Lambda_2 = \frac{\lambda_{feff}[\varepsilon\rho_f(c_p)_f+(1-\varepsilon)\rho_s(c_p)_s]}{\varepsilon\rho_f(c_p)_f[\lambda_{feff}+\lambda_{seff}]}$$

and

$$\delta = \frac{1}{h_{sf}a_{sf}} \frac{\varepsilon[\rho_f(c_p)_f]^3 v_f^2}{[\varepsilon\rho_f(c_p)_f+(1-\varepsilon)\rho_s(c_p)_s](\lambda_{feff}+\lambda_{seff})}.$$

ANALYSIS

For the dimensionless fluid temperature, Θ_f, initial and boundary conditions discussed in the beginning of the previous section can be written in the following form:

$$\Theta_f(\xi_1,\xi_2,0) = \Theta_0 \tag{4}$$

$$\Theta_f(0,\xi_2,\tau) = 1, \quad \frac{\partial\Theta_f}{\partial\xi_1}(R_1,\xi_2,\tau) = 0 \tag{5}$$

$$\Theta_f(\xi_1,0,\tau) = 0, \quad \Theta_f(\xi_1,R_2,\tau) = 0 \tag{6}$$

where

$$\Theta_0 = \frac{T_0-T_w}{T_{in}-T_w}$$

and can be either positive, negative or zero,

$$R_1 = \frac{\rho_f(c_p)_f v_f}{\lambda_{feff}+\lambda_{seff}} L_1 \quad \text{and} \quad R_2 = \frac{\rho_f(c_p)_f v_f}{\lambda_{feff}+\lambda_{seff}} L_2.$$

The zero temperature gradient at the fluid outlet boundary, $\xi_1 = R_1$, means that it is assumed that the temperature of the fluid does not change after the fluid left the packed bed and there is no temperature jump at the outlet boundary.

The solution of equation (2) with initial conditions of equation (4) and boundary conditions of equations (5)–(6) cannot be obtained simply as a product of the solutions of the pertinent 1-D problems. Following ref. [13], this problem is reduced to two simpler problems: a problem of steady temperature and a problem with zero surface temperature/heat flux

$$\Theta_f(\xi_1,\xi_2,\tau) = u(\xi_1,\xi_2)+w(\xi_1,\xi_2,\tau). \tag{7}$$

The function u satisfies the following steady equation

$$\frac{\partial u}{\partial\xi_1} = \frac{\partial^2 u}{\partial\xi_1^2}+\frac{\partial^2 u}{\partial\xi_2^2} \tag{8}$$

and the boundary conditions (5) and (6). The function w satisfies the transient equation (2) with boundary conditions (6) at the walls, the following zero boundary conditions at the fluid inlet and outlet

$$w(0,\xi_2,\tau) = 0, \quad \frac{\partial w}{\partial\xi_1}(R_1,\xi_2,\tau) = 0 \tag{9}$$

and the following initial condition

$$\Theta_f(\xi_1,\xi_2,0) = \Theta_0-u(\xi_1,\xi_2). \tag{10}$$

The solution for the function u is obtained using the classical Fourier method as

$$u = \sum_{n=1}^{\infty} C_n \exp\left(\frac{\xi_1}{2}\right)\sin\left(\frac{\pi n}{R_2}\xi_2\right)\left[-\left(\frac{1}{2}+b_n\right)\times\exp[b_n(R_1-\xi_1)]+\left(\frac{1}{2}-b_n\right)\times\exp[-b_n(R_1-\xi_1)]\right] \tag{11}$$

where

$$b_n = \left[\frac{1}{4}+\left(\frac{\pi n}{R_2}\right)^2\right]^{1/2} \tag{12}$$

and the series coefficients in equation (11) are

$$C_n = \frac{2[1-(-1)^n]}{\pi n[-(\frac{1}{2}+b_n)\exp(b_nR_1)+(\frac{1}{2}-b_n)\exp(-b_nR_1)]}. \tag{13}$$

The solution for the function w is also obtained using the classical Fourier method as

$$w = \sum_{n,m=1}^{\infty} D_{n,m} \exp\left\{\frac{\xi_1}{2}-\left[\frac{1}{4}+a_m^2+\left(\frac{\pi n}{R_2}\right)^2\right]\tau\right\}\times\sin(a_m\xi_1)\sin\left(\frac{\pi n}{R_2}\xi_2\right) \tag{14}$$

where a_m are different positive solutions of the transcendental equation

$$\tan(a_mR_1) = -2a_m \tag{15}$$

and the series coefficients in equation (14) are

$$D_{n,m} = \frac{[1-(-1)^n]\left\{\Theta_0\frac{R_2}{\pi n}\frac{4a_m}{1+4a_m^2}-\frac{a_mL}{\pi n(a_m^2+b_n^2)}\right\}}{\frac{R_2}{2}\left[\frac{R_1}{2}-\frac{\sin(2a_mR_1)}{4a_m}\right]}. \tag{16}$$

The solution for the dimensionless fluid temperature, Θ_f, now follows from equations (7), (11) and (14).

Figure 2 shows a typical space distribution of the dimensionless fluid temperature. For this particular calculation the temperature at the walls of the packed bed is chosen to be between the initial and inlet tem-

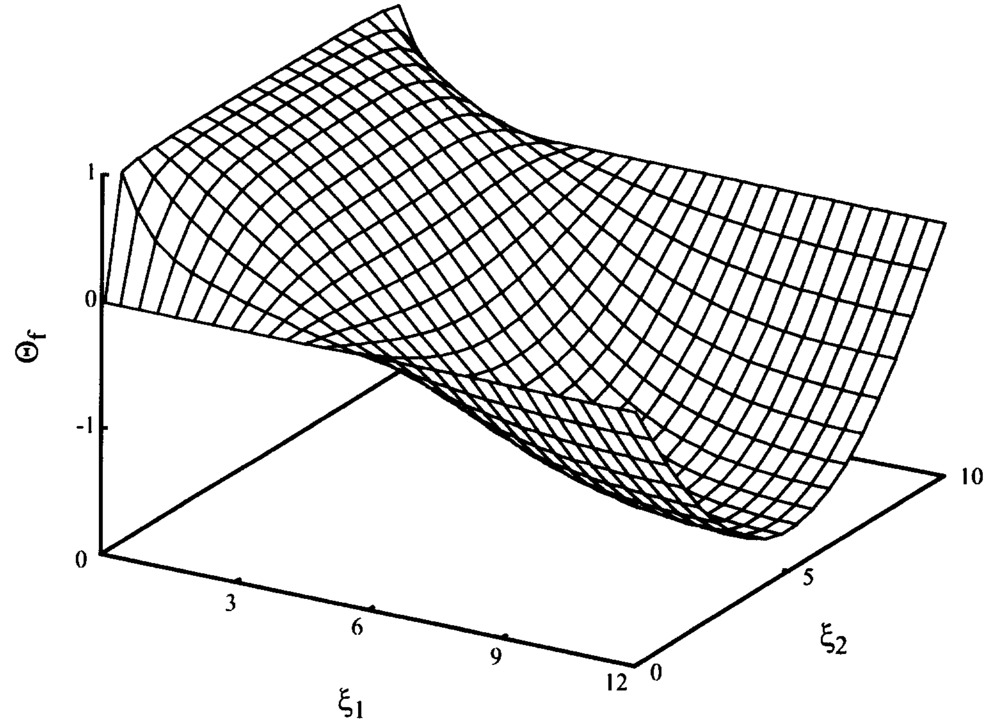

Fig. 2. The space distribution of the dimensionless fluid temperature, Θ_f, for: $\Theta_0 = -2$, $R_1 = 12$, $R_2 = 10$, $\Lambda_1 = 13$ and $\Lambda_2 = 2.6$ for $\tau = 4$. These values of Λ_1 and Λ_2 correspond to the following ratios of the thermophysical properties: $(c_p)_f\rho_f = 0.25(c_p)_s\rho_s$, $\lambda_{feff} = 0.25\lambda_{seff}$ and $\varepsilon = 0.25$.

peratures. According to equation (1) the difference between the fluid and solid dimensionless temperatures is of the order of the small parameter δ.

The dimensionless temperature difference between the solid and fluid phases can be now found from equations (3), (7), (11) and (14) as

$$\Delta\Theta = (\Lambda_1 - \Lambda_2) \times \left[\frac{u}{2} + \sum_{n=1}^{\infty} b_n C_n \exp\left(\frac{\xi_1}{2}\right)\sin\left(\frac{\pi n}{R_2}\xi_2\right) \times \left\{\left(\frac{1}{2}+b_n\right)\exp\left[b_n(R_1-\xi_1)\right] + \left(\frac{1}{2}-b_n\right)\exp\left[-b_n(R_1-\xi_1)\right]\right\}\right] - (1-\Lambda_2)\sum_{n,m=1}^{\infty}\left[\frac{1}{4}+a_m^2+\left(\frac{\pi n}{R_2}\right)^2\right]D_{n,m} \times \exp\left\{\frac{\xi_1}{2}-\left[\frac{1}{4}+a_m^2+\left(\frac{\pi n}{R_2}\right)^2\right]\tau\right\} \times \sin(a_m\xi_1)\sin\left(\frac{\pi n}{R_2}\xi_2\right) + (\Lambda_1-\Lambda_2) \times \left[\frac{w}{2} + \sum_{n,m=1}^{\infty} a_m D_{n,m}\exp\left\{\frac{\xi_1}{2}-\left[\frac{1}{4}+a_m^2+\left(\frac{\pi n}{R_2}\right)^2\right]\tau\right\} \times \cos(a_m\xi_1)\sin\left(\frac{\pi n}{R_2}\xi_2\right)\right] \quad (17)$$

where the coefficients b_n, C_n, a_m and $D_{n,m}$ are determined by equations (12)–(13) and (15)–(16) and the functions u and w are given by equations (11) and (14) correspondingly.

Figures 3(a)–(b) depict the space distributions of the dimensionless temperature difference. It can be seen that the temperature difference consists of the steady and transient components. The steady component comes from the steady solution for the function u and describes the temperature difference at $\tau = \infty$. Unlike the results presented in ref. [12], the temperature difference for this problem does not approach zero when time approaches infinity. This is because in our case the fluid inlet temperature differs from the temperature at the walls of the packed bed. The steady component is localized near the inlet boundary. The transient component comes from the transient solution for the function w and describes a wave propagating in the ξ_1-direction from the fluid inlet boundary. The amplitude of the wave quickly decreases while the wave propagates downstream.

CONCLUSIONS

The process of heating a 2-D porous packed bed by a non-thermal equilibrium flow of incompressible fluid is investigated. Using the perturbation technique, the analytical solution for the problem is obtained. It is shown that the temperature difference between the fluid and solid phases consists of the steady and tran-

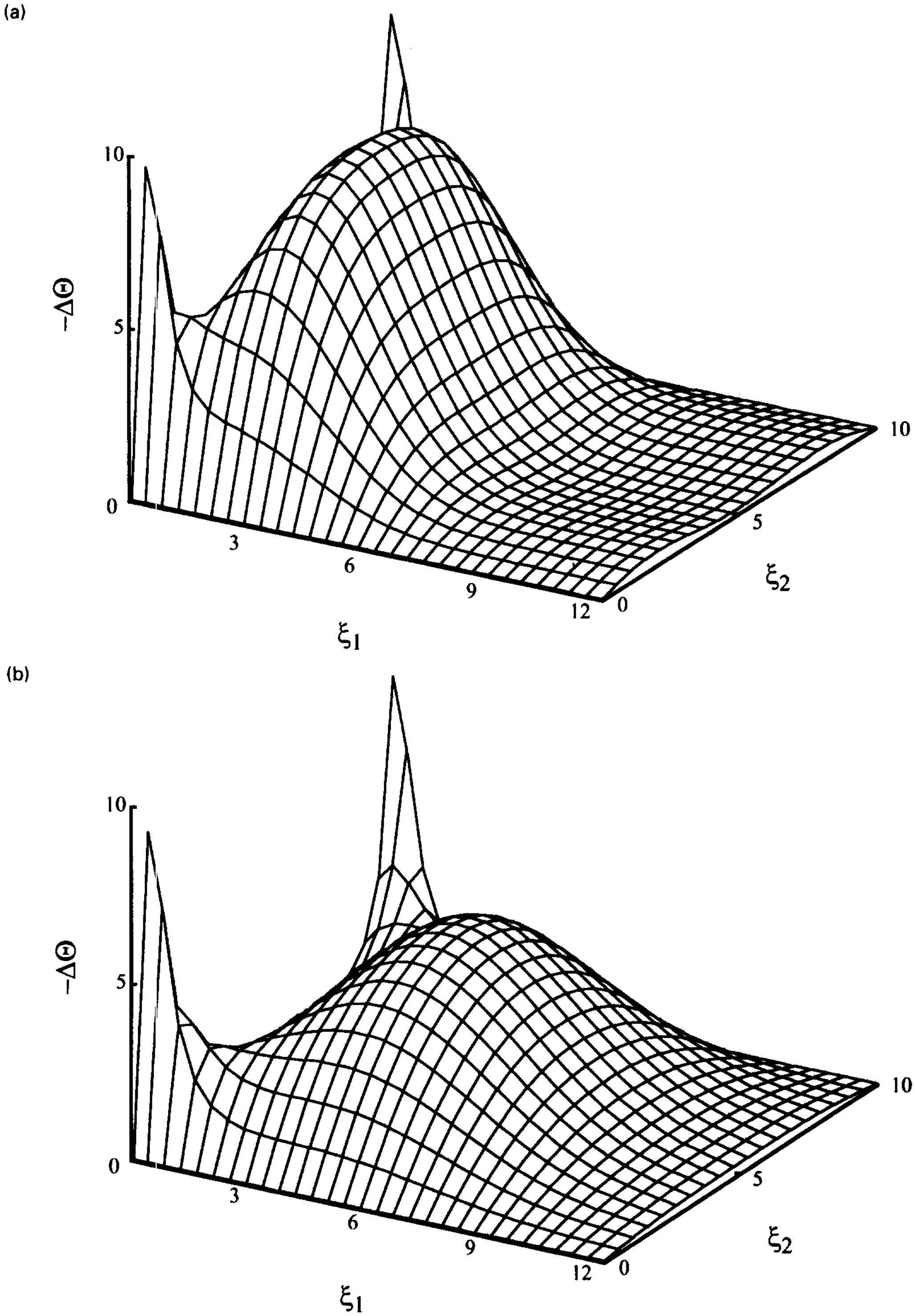

Fig. 3. The space distributions of the dimensionless temperature difference $-\Delta\Theta$, for $\Theta_0 = -2$, $R_1 = 12$, $R_2 = 10$, $\Lambda = 13$ and $\Lambda_2 = 2.6$ for the following moments of the dimensionless time: (a) $\tau = 2$ (b) $\tau = 4$.

sient components. The steady component describes the temperature difference at $\tau = \infty$—it is localized near the inlet boundary. The transient component describes a wave propagating in the ξ_1-direction from the fluid inlet boundary. The amplitude of the wave quickly decreases while the wave propagates downstream.

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